Semi-empirical Modeling of Charge and Discharge Profiles for a LiCoO$_2$ Electrode

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To be submitted to Journal of Power Source as a short communication

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Abstract

The experimental charge and discharge profiles of a LiCoO$_2$ electrode show that the overpotential of the electrode does not change much during galvanostatic charge, but changes significantly during galvanostatic discharge. Semi-empirical porous electrode models are presented to simulate the charge and discharge profiles of the LiCoO$_2$ electrode. The symmetry factor is empirically assumed to decrease with the state of discharge of the electrode to enable the model predictions to agree well with the experimental discharge profiles.

Keywords: Lithium cobalt oxide electrode; LiCoO$_2$; numerical simulation; symmetry factor.
Introduction

Lithium cobalt oxide has been widely used as cathode material in lithium ion secondary batteries. The reaction of LiCoO$_2$ is well known to be lithium ion extraction from and insertion into a layered cobalt dioxide matrix. Many researchers [1-3] have simulated the charge and discharge behaviors of a full cell sandwich which uses LiCoO$_2$ as cathode material. However, very few studies can be found in the literature focusing on LiCoO$_2$ electrode alone. In this work, the rate capability of a LiCoO$_2$ electrode was studied in half cells with a three-electrode setup. The study shows that the overpotential of the LiCoO$_2$ electrode does not change much during galvanostatic charge, but changes significantly with cell state of discharge during galvanostatic discharge. A pseudo-2D porous electrode model [4-6] was used to simulate charge and discharge profiles of the LiCoO$_2$ electrode. Empirical expressions were used in the Butler Volmer equation to enable the model predictions to agree well with the experimental data.

Experiment

The rate capability of a LiCoO$_2$ electrode was measured using a Swagelok-type half cell (see Fig. 1). The half cell consisted of a LiCoO$_2$ working electrode, a separator and a lithium foil counter electrode. Cone shaped lithium metal with a sharp end was placed just above the LiCoO$_2$ electrode to serve as the reference electrode. The potential of the LiCoO$_2$ electrode vs Li/Li$^+$ was measured directly in the experiment. The working electrode was a round disc with a diameter of 1/2 inch pouched out of a singled-sided LiCoO$_2$ sheet electrode provided by Mine Safety Appliances (Sparks, MD). The Celgard-2300 polypropylene membrane (Charlotte, NC) of a thickness of 25 µm was used as the separator. The electrolyte (Ferro, Independence, OH) was 1.0M LiPF$_6$ in a solvent...
mixture of EC/PC/EMC/DEC. The half cell was assembled in an argon-filled glovebox, removed and placed in a Tenney environmental chamber controlled at 15 °C. An 8-channel Arbin battery test unit was used to conduct the rate capability test.

The half cell was first cycled several times at the C/13 rate (C=4 mA) between 3.0-4.35 V vs Li/Li⁺ to stabilize the electrochemical performance of the LiCoO₂ electrode. The capacity of the LiCoO₂ disc electrode was measured around 4mAha when it was cycled between the given voltage window. The currents used in rate capability test included 6×10⁻⁵, 3×10⁻⁴, 6×10⁻⁴, 1.5×10⁻³ and 3×10⁻³ A, which roughly correspond to the C/66, C/13, C/7, C/2.7, and C/1.3 rates, respectively. For rate capability test at C/66 rate, a single stage constant current protocol was used to charge and discharge the half cell. For other rates, a two stage constant current protocol was used. The cell was first charged or discharged to the desired voltage limit (3.0 or 4.35 V) using the target rate. Then a small current (C/66 rate) was applied to continue charge or discharge until the desired voltage limit was reached again. The use of the two stage charge/discharge protocol was to ensure the cell to reach the same SOC at the beginning of charge or discharge for all tests.

The charge and discharge profiles of the LiCoO₂ electrode are presented in Figs. 2-3 where the potential is plotted against x in LiₓCoO₂. The value of x in LiₓCoO₂ was calculated by using the following formula:

\[ x = x₀ ± \frac{Q}{(0.274 \times W)} \]  

where \( W \) was the active material loading of the LiCoO₂ disc electrode and was determined to be 0.0245g from disc electrode size and loading data provided by manufacture. The factor of 0.274 was the theoretical capacity in Ah to convert 1g LiCoO₂
completely to CoO₂. The accumulative charge or discharge capacity $Q$ in Ah was obtained directly from experimental data. The initial state ($x$ in Li$_x$CoO$_2$) of the LiCoO$_2$ electrode was determined to be 0.99 for charge and 0.393 for discharge from low rate potential profiles.

The potential profiles of the LiCoO$_2$ electrode measured during the first stage charge process are compared with those measured during the first stage discharge process in Fig. 4 for both the C/66 and C/1.3 rates. As seen in Fig. 4, the potential profile measured during charge at the C/66 rate almost overlays with the potential profile measured during discharge at the same rate, which indicates that the electrode is very close to equilibrium. Unfortunately, this is not true for the C/1.3 rate. Several observations can be made by comparing the C/66 and C/1.3 rate charge and discharge profiles: First, the overpotential (relative to the equilibrium potential determined by averaging the C/66 rate potentials measured in both charge and discharge) does not change much during galvanostatic charge for both the C/66 and C/1.3 rates. But it exhibits heavy dependence on state of discharge (SOD) during discharge. Second, the potential of the LiCoO$_2$ electrode starts to drop at high SOD when the discharge rate is low. But it happens at low SOC if discharge rate is high. Third, the potential plateau around $x = 0.5$ (in Li$_x$CoO$_2$) gradually disappears in both charge and discharge profiles when the current is increased. Many researchers [7-9] explained the potential plateau in that region by a phase transition or equilibrium between ordered and disordered lithium ions in the CoO$_2$ framework. The movement of large amounts of lithium ions in a short time at high current rates may prevent this phase equilibrium from occurring, which would explain the gradual disappearance of the potential plateau at $x = 0.5$. And lastly,
the LiCoO₂ electrode does not exhibit good rate capability, because it reaches the cutoff voltage early at low cell SOD when current rate is relatively high (C/1.3).

**Model**

Porous electrode models [4-6] have been heavily used in the literature to study the performance of lithium ion battery systems. A detailed explanation about porous electrode model used here can be found elsewhere [5-6]. The model equations are summarized below for the convenience of the readers.

The mass transport in solid LiCoO₂ particles is described by using Fick’s diffusion law:

\[
\frac{\partial c_s}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( D_s r^2 \frac{\partial c_s}{\partial r} \right)
\]

\[
-D_s \frac{\partial c_s}{\partial r} \bigg|_{r=0} = 0
\]

\[
-D_s \frac{\partial c_s}{\partial r} \bigg|_{r=r_s} = \frac{j}{F}
\]

A mass balance on the lithium ions in the solution phase yields:

\[
\frac{\partial \epsilon c_e}{\partial t} - \nabla \cdot \left( D_{seff} \nabla c_e \right) - \frac{1-T_e^0}{F} a_s j = 0
\]

where the specific interfacial area of the electrode is \( a_s = \frac{3 \epsilon_s}{R_s} \).

The potential distributions in solid and solution phases are described as follows:

\[
\nabla \cdot (\sigma_s \nabla \phi_s) - a_s j = 0
\]

\[
\nabla \left( \kappa_{seff} \nabla \phi_s - \frac{2RTK_{seff}}{F} \left( 1 - T_e^0 \right) \left( 1 + \frac{d \ln f_s}{d \ln c_e} \right) \nabla \ln c_e \right) + a_s j = 0
\]
The concentrations and potentials in the solid and solution phase are coupled together through the intercalation current density \( j \) which is calculated through the Butler Volmer equation:

\[
j = i_{0,p} \left( \exp \left( \frac{\alpha_e F}{RT} (\phi_e - \phi - U_{eq} - jR_f) \right) - \exp \left( -\frac{\alpha_e F}{RT} (\phi_e - \phi - U_{eq} - jR_f) \right) \right) \quad (6, a)
\]

where the exchange current density \( i_{0,p} \) is formulated as:

\[
i_{0,p} = k_p \epsilon_e^0 \left( c_{s,\text{max}} - c_s \bigg|_{x=R_s} \right) \alpha_e c_s \bigg|_{x=R_s}^{a_e} \quad (6, b)
\]

The computation schematic of a half cell sandwich is shown in Fig. 5. The boundary conditions are thus given as:

At \( x = 0 \):

\[
-D_{e,\text{pos}}^{\text{eff}} \frac{\partial c_e}{\partial x} = 0, \quad -\sigma_{s,\text{pos}} \frac{\partial \phi}{\partial x} = \frac{I}{S_p}, \quad -\kappa_{e,\text{pos}}^{\text{eff}} \frac{\partial \phi}{\partial x} = 0 \quad (7, a-c)
\]

At \( x = \delta_p \):

\[
-D_{e,\text{sep}}^{\text{eff}} \frac{\partial c_e}{\partial x} \bigg|_{x=\delta_p} = -D_{e,\text{sep}}^{\text{eff}} \frac{\partial c_e}{\partial x} \bigg|_{x=\delta_p}, \quad -\sigma_{s,\text{pos}}^{\text{eff}} \frac{\partial \phi}{\partial x} \bigg|_{x=\delta_p} = 0, \quad \text{and} \quad -\kappa_{e,\text{sep}}^{\text{eff}} \frac{\partial \phi}{\partial x} \bigg|_{x=\delta_p} = 0 \quad (8, a-c)
\]

At \( x = \delta_p + \delta_i \):

\[
\frac{I}{S_{sep}^{\text{eff}}} (1-i_0^0) = -D_{e,\text{sep}}^{\text{eff}} \nabla c_e \quad (9, a-b)
\]

\[
\frac{I}{S_{sep}^{\text{eff}}} = -i_{0,n} \left[ \exp \left( \frac{0.5F}{RT} (0-\phi_e) \right) - \exp \left( -\frac{0.5F}{RT} (0-\phi_e) \right) \right]
\]

The model equations (Eqs. 2-9) were discretized in space using finite difference and solved using the DASSL subroutine in Fortran.
The properties of the LiPF$_6$ based electrolyte, such as diffusion coefficient and conductivity are important model parameters. But no experimental data were found in the literature for the electrolyte system used in this study (1.0M LiPF$_6$ in the EC/PC/EMC/DEC mixture). Valøen and Reimers [10] measured the properties of LiPF$_6$ as a function of temperature and concentration in a slightly different electrolyte system which is LiPF$_6$ in a PC/EC/DMC mixture. Their results were used here in the simulation assuming that LiPF$_6$ behaves similarly in those two electrolytes. The diffusion coefficient of the electrolyte was found to be [10]:

$$\log(D_e) = -4.43 - 54 / (T - 5 \times 10^3 c_e - 229) - 0.22 \times 10^3 c_e$$  \hspace{1cm} (10, a)$$

The expression for the conductivity of the electrolyte was found to be [10]:

$$\kappa_e = c_e \left( -10.5 + 0.074 T - 6.96 \times 10^{-5} T^2 + 668 c_e \right) \left( -17.8 c_e T + 0.028 c_e T^2 + 4.94 \times 10^5 c_e^2 - 886 c_e^2 T \right)$$  \hspace{1cm} (10, b)$$

The effective value of the diffusion coefficient and conductivity of the electrolyte were used in the simulation to account for the actual path length of the species in porous media. They were calculated through Bruggeman’s correlation:

$$D_e^{\text{eff}} = D_e e^{\text{Brug}}$$  \hspace{1cm} (11, a-b)$$

$$\kappa_e^{\text{eff}} = \kappa_e e^{\text{Brug}}$$

The thermodynamic factor which account for the non-ideality of the electrolyte was found to be [10]:

$$v = (1 - t^0_e) \left( 1 + \frac{d \ln f_e}{d \ln c_e} \right)$$  \hspace{1cm} (12)$$

$$= 0.601 - 7.59 c_e^{0.5} + 3.1 \times 10^4 (2.53 - 0.0052 T) c_e^{1.5}$$

Simulation
1. Simulation on charge profiles of the LiCoO$_2$ electrode

Fig. 6 presents a comparison of the experimental data with the predicted charge curve using model equations presented above. Clearly, the model predictions do not agree with the experimental data for $x \geq 0.8$. The model parameters used in the simulation are listed in Table 1. The exchange current density $i_{0,p}$ expression and the flat open circuit potential (OCP) of LiCoO$_2$ from $x = 0.75 \sim 1$ was found to be the combined cause of lack of agreement. Fig. 7 shows that the change of $i_{0,p}$ with $x$ during the simulation where it can be seen that $i_{0,p}$ has a small value at the beginning of charge because $c_{s,\text{max}} - c_s \big|_{\nu=R_s}$ is close to zero, which causes the Butler Volmer equation to predict a large overpotential for the set current density. As $i_{0,p}$ gradually increases during charge because $c_{s,\text{max}} - c_s \big|_{\nu=R_s}$ is becoming larger, the Butler Volmer equation predicts a smaller overpotential for the same current density. The exchange current density in Eq. 6b was derived based on the assumption of a close relation with the Nernstian form of the OCP [4]. However, the OCP of LiCoO$_2$ around $x \approx 0.75 \sim 1$ is highly non-Nernstian because it shows weak dependence on the lithium ion concentration in the solid matrix (see low rate profiles in Fig. 4). To address this problem, the concentration of empty sites $c_{s,\text{max}} - c_s \big|_{\nu=R_s}$ at the surface of solid particle is removed from exchange current density expression and the $i_{0,p}$ expression becomes:

$$i_{0,p} = k_p c_e^{\alpha_e} c_s \big|_{\nu=R_s}^{\alpha_s}$$  \hspace{1cm} (13)
Eq. 13 means that the charge kinetics has a relatively strong dependence on the concentration of \( Li^+ \) occupied sites in \( LiCoO_2 \) but week (or no) dependence on the concentration of empty site, similar to kinetics in desorption process.

Fig. 8 compares the experimental and simulated charge profiles of the \( LiCoO_2 \) electrode using Eq. 13 for \( i_{0,p} \) instead of Eq. 6b. Model parameters used in the simulation are listed in Table 2. Simulation results show good agreement with experiment ones for all charge rates. This study shows that with only constant model parameters, the model could predict satisfactorily the charge profiles of the \( LiCoO_2 \) electrode where overpotential does not vary much with SOC. The model could not predict the disappearance of the voltage plateau around \( x = 0.5 \) at high current rates because it does not include any mechanism to account for the phenomena.

2. Simulation on discharge profiles of the \( LiCoO_2 \) electrode

Fig. 9 shows the simulation results when the model parameters are kept constant during discharge using Eq. 6b for \( i_{0,p} \). The parameters used in the simulation are listed in Table. 3. The simulated results fit the experiment profiles well at the beginning of discharge and when current rates are low. But the model failed to predict the increased potential drop with cell SOD at high rates. Extensive simulations indicate that it is very difficult for the model to predict experimental discharge profiles when all model parameters are kept constant except those for the electrolyte (Eqs 10 and 12).

In order to achieve a better fit on discharge profiles, an empirical expression was introduced to the kinetic expression and used along with porous electrode model. The symmetry factor of the \( Li^+ \) insertion reaction is assumed to decrease with SOD based on
our observation of increased potential drop during galvanostatic discharge. The Li$^+$ insertion reaction is usually considered as one electron transfer process.

\[ \text{Li}^+ + S + e^- \xrightarrow{} \text{Li} - S \] (14)

The symmetry factor represents the fractional charge that promotes the cathodic reaction [4]. When symmetry factor $\beta$ decreases with SOD, it physically implies that the reversible lithium ion intercalation reaction favors more the anodic reaction (Li oxidation) than it does the cathodic reaction (Li$^+$ reduction). That is, it becomes kinetically less favorable to insert Li$^+$ into the electrode as discharge goes on, which requires increased driving force, kinetic overpotential, to maintain the galvanostatic discharge current forced by the external circuit. The empirical expression for the symmetry factor $\beta$ used is as follows:

\[
\beta = 0.5 \left(1 - \frac{1}{1 + \exp\left(a \left( b - c_s \right) / c_{s,\text{max}}\right)}\right) \tag{15, a-c}
\]

\[
\alpha_c = \beta
\]

\[
\alpha_a = 1 - \beta
\]

Fig. 10 shows the simulation predicted potential profiles during the discharge of the LiCoO$_2$ electrode at different rates. The solid lines represent the simulated profiles and the symbols represent experimental data. Model parameters used to obtain Fig. 10 are listed in Table 4. Simulation results show good agreement with experimental data for all conditions. There is a significantly improvement over the previous case (Fig. 9) when the symmetry factor is allowed to decrease with SOD. The model well captures the fine details of experiment profiles at the beginning of discharge. The discharge profile of the
LiCoO₂ electrode is a straight line before $x$ reaches 0.5 at C/66 rate. That straight line gradually turns into a curve with increased curvature at high rate, which could be an indication of increased solid phase diffusion limitation in the LiCoO₂ electrode. The model well predicts the shape change of the experimental profiles at the beginning of discharge using a relatively small solid phase diffusion coefficient. It does not predict the disappearance of voltage plateau around $x = 0.5$ at high rates for the same reason discussed earlier.

In order to use the discharge model in other simulations, the parameters $a$ and $b$ in symmetry factor expression need to be correlated to current rates. The values of $a$ and $b$ obtained from fitting experimental discharge profiles are plotted against current rate in Fig. 11 and 12 respectively. Parameter $a$ is found to depend rationally on current rate and parameter $b$ linearly. The correlation expressions can be found in Table 5. The model now could be used to predict discharge profiles of the LiCoO₂ electrode given a discharge current rate. It should be noted that the treatment offered in the work, the dependence of symmetry factor on the surface concentration, is empirical. Nonetheless, the semi-empirical model should yield sufficient good results for electrode analysis provided that the operation conditions of the electrode are not varied to the extent that the empirical representation becomes substantially inaccurate.

3. Simulations using the models for LiCoO₂ electrode

In previous sections, porous electrode models were used to simulate experimental charge and discharge profiles of a LiCoO₂ electrode, which is the main purpose of the work. It is shown that simulation results agree well with experimental ones. With the
availability of accurate models, one can study the electrochemical behaviors of the LiCoO₂ electrode in details.

The charge and discharge profiles of the LiCoO₂ electrode are simulated under different current rates and the results are shown in Fig. 13. The voltage window for the simulation is 3.0 to 4.35V. The rate capacities are compared to the experiment values in Fig. 14. Simulation shows that the LiCoO₂ electrode used in the study does not exhibit a good rate capability in discharge. The model predicts the electrode can only achieve about 75% of the total capacity at 1C rate and 50% at 2C rate.

The rate capacity data can be converted into energy-power data to construct a Ragone plot which is a useful tool in comparing different battery designs. It is very time-consuming to construct such a plot experimentally. However, computer simulations can easily and rapidly generate a significant number of data points for a Ragone plot. The Ragone plot for the LiCoO₂ electrode is given in Fig. 15. The specific energy (Wh/m²) and power (W/m²) are higher than normal values because the electrode is charged to a higher SOC (\( x \approx 0.4 \)) which could be the most direct way to increase specific energy and power. But in real-world applications, the LiCoO₂ electrode is rarely charged to such a high SOC because of the negative effect of phase transition at \( x \approx 0.5 \) on the life span of the electrode. The high end of charge voltage is used in the rate capability test to help us identify the solid phase diffusion limitation in the LiCoO₂ electrode from the shape change of discharge profiles. Ragone plot also shows that the electrode is not suited for applications where high rate discharge is needed because the specific energy of the electrode deteriorates quickly after the current passes 2C.
The calculated salt concentration and solution phase potential profiles at 2C rate are shown in Fig. 16 and 17 respectively. The profiles of the salt concentration show moderate concentration polarization. And diffusion coefficient of electrolyte is around $2.5 \times 10^{-6}$ cm$^2$/s which is several orders of magnitude higher than solid phase diffusion coefficient. The solution phase potential drop across the cell remains near 35 mV throughout the discharge. So neither concentration polarization nor solution phase potential drop substantially limits the cell performance. The solid phase concentration profiles within particles at the electrode-electrolyte interface are shown in Fig. 18 for 2C discharge process. The large concentration gradient inside the particle at the end of discharge indicates strong solid phase diffusion limitation existing in the electrode.

**Conclusion**

Rate capability study on a LiCoO$_2$ electrode shows that the discharge behaviors of the LiCoO$_2$ electrode are quite different from its charge behaviors. The overpotential of the electrode in galvanostatic discharge depends strongly on cell SOD, but it does not vary much in galvanostatic charge.

A pseudo-2D porous electrode model is first used to simulate the charge profiles of the LiCoO$_2$ electrode. It is found that the model predicts false potential profiles at the beginning of charge, which is corrected by assuming charge kinetics does not depend on the surface concentration of empty sites in LiCoO$_2$ particle, similar as desorption kinetics. With only constant model parameters, the model predicts potential profiles in good agreement with experimental ones.

Extensive simulations show that the pseudo-2D model has difficulty to predict experimental discharge profiles of the LiCoO$_2$ electrode when model parameters are kept
constant. Therefore, symmetry factor in Butler-Volmer equation is empirically assumed to decrease with cell SOD to account for increased potential drop during galvanostatic discharge. It means that the insertion of Li\(^+\) into LiCoO\(_2\) electrode becomes kinetically less favorable during discharge, which requires an increased overpotential to maintain the constant discharge current forced by the external circuit. Simulation results from the semi-empirical discharge model are significantly improved and match to experimental discharge profiles well. The model should yield sufficient good results for electrode analysis given that operation conditions of the electrode are not varied to the extent that the empirical representation becomes substantially inaccurate.

**Acknowledgement**

The authors are grateful for the financial support of the project by the National Reconnaissance Office (NRO) under contract # NRO-000-03-C-0122.
List of symbols

- \( a_s \) Specific interfacial area of the electrode, cm\(^2\)/cm\(^3\)
- \( \text{Brug} \) Bruggeman coefficient
- \( c_e \) Li\(^+\) concentration in the electrolyte, mol/cm\(^3\)
- \( c_s \) Solid phase Li\(^+\) concentration in the LiCoO\(_2\) electrode, mol/cm\(^3\)
- \( c_{s,\text{max}} \) Maximum solid phase Li\(^+\) concentration in the LiCoO\(_2\) electrode, mol/cm\(^3\)
- \( D_e \) Diffusion coefficient of the electrolyte, cm\(^2\)/s
- \( D_s \) Solid phase diffusion coefficient, cm\(^2\)/s
- \( f_\pm \) Mean molar salt activity coefficient
- \( F \) Faraday’s constant, 96487 C/mol
- \( i_0 \) Exchange current density, A/cm\(^2\)
- \( I \) Applied current, A
- \( j \) Intercalation current density, A/cm\(^2\)
- \( k_p \) Kinetic rate constant, (A/cm\(^2\))/(mol/cm\(^3\))\(^{1.5}\)
- \( Q \) Charge or discharge capacity, Ah
- \( R \) Gas constant, 8.3145 J/mol·K
- \( R_f \) Contact resistance, Ω·cm\(^2\)
- \( R_s \) Particle radius of the LiCoO\(_2\) electrode, cm
- \( S \) Geometric area of the electrode, cm\(^2\)
- \( t \) Time, s
- \( t_{\text{tr}}^0 \) Transference number of the electrolyte
$T$ Temperature, K

$U_{eq}$ Equilibrium potential of the electrode, V

$v$ Thermodynamic factor of the electrolyte

$W$ Active material loading in the electrode, g

$x$ $x$ in Li$_x$CoO$_2$, SOC of the electrode

Greek

$\alpha_a, \alpha_c$ Transfer coefficients

$\phi_e, \phi_s$ Liquid or solid phase potential, V

$\delta_p, \delta_s$ Electrode or separator thickness, cm

$\epsilon_e, \epsilon_s$ Volume fraction of the electrolyte or active material in solid phase

$\beta$ Symmetry factor

$\kappa_e$ Conductivity of the electrolyte, S/cm

$\sigma_s$ Conductivity of the solid phase, S/cm

Subscripts or superscripts

$c, d$ Charge or discharge

$eff$ Effective value

$n, p$ Positive or negative electrode

$sep$ Separator
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Fig. 3 Experimental galvanostatic discharge profiles of the LiCoO₂ half-cell (C=4mA).

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Fig. 9 The best simulation results (lines) obtained when model parameters are constants. The rates from top to bottom are C/66, C/13, C/7, C/2.7 and C/1.3 (C=4 mA).

Fig. 10 Comparison of experiment (symbols) and simulation (lines) discharge profiles at different current rates. Symmetry factor $\beta$ is assumed to decrease with cell SOD in galvanostatic discharge.

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<td>&lt;i&gt;ɛ&lt;/i&gt;&lt;sub&gt;s&lt;/sub&gt;</td>
<td>0.60&lt;sup&gt;a&lt;/sup&gt;</td>
<td>R&lt;sub&gt;y&lt;/sub&gt; (Ω·cm&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>200&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>σ&lt;sub&gt;y&lt;/sub&gt; (S/cm)</td>
<td>0.1&lt;sup&gt;b&lt;/sup&gt;</td>
<td>k&lt;sub&gt;p,c&lt;/sub&gt; (A/cm&lt;sup&gt;2&lt;/sup&gt;/(mol·cm&lt;sup&gt;3&lt;/sup&gt;)&lt;sup&gt;1.5&lt;/sup&gt;)</td>
<td>4.2×10&lt;sup&gt;-2&lt;/sup&gt;&lt;sup&gt;c&lt;/sup&gt;</td>
</tr>
<tr>
<td>t&lt;sub&gt;y&lt;/sub&gt;&lt;sup&gt;0&lt;/sup&gt;</td>
<td>0.363&lt;sup&gt;b&lt;/sup&gt;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Separator</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c&lt;sub&gt;e,0&lt;/sub&gt;</td>
<td>1×10&lt;sup&gt;-3&lt;/sup&gt;&lt;sup&gt;a&lt;/sup&gt;</td>
<td>S&lt;sub&gt;sep&lt;/sub&gt; (cm&lt;sup&gt;2&lt;/sup&gt;)</td>
<td>1.267&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>δ&lt;sub&gt;s&lt;/sub&gt; (cm)</td>
<td>25×10&lt;sup&gt;-4&lt;/sup&gt;&lt;sup&gt;a&lt;/sup&gt;</td>
<td>&lt;i&gt;ɛ&lt;/i&gt;&lt;sub&gt;c&lt;/sub&gt;</td>
<td>0.46&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>D&lt;sub&gt;e&lt;/sub&gt;</td>
<td>Eq. 10a</td>
<td>&lt;i&gt;κ&lt;/i&gt;&lt;sub&gt;e&lt;/sub&gt;</td>
<td>Eq. 10b</td>
</tr>
<tr>
<td>(∁&lt;sub&gt;y&lt;/sub&gt;&lt;sup&gt;0&lt;/sup&gt;)&lt;sup&gt;2&lt;/sup&gt;</td>
<td>Eq. 12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Manufacture data or experiment data

<sup>b</sup> From references<sup>1-3</sup>

<sup>c</sup> Values fit to experiment data
Table 2. Model parameters used to obtain Fig. 8 using Eq. 13

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{p,c}$ (A/cm²/(mol·cm³))</td>
<td>$6.0 \times 10^{-3}$</td>
<td>$x_{0,p}$</td>
<td>0.99</td>
</tr>
</tbody>
</table>

* Refer to Table 1 for parameters not listed here.

$^c$ Values fit to experiment data

Table 3. Model parameters used to obtain Fig. 9* using Eq. 6b

<table>
<thead>
<tr>
<th>Parameter</th>
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<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$D_s$ (cm²/s)</td>
<td>$1.4 \times 10^{-10}$</td>
<td>$x_{0,p}$</td>
<td>0.393</td>
</tr>
<tr>
<td>$k_{p,d}$ (A/cm²)/(mol/cm³)$^{1.5}$</td>
<td>0.25</td>
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</tbody>
</table>

* Refer to Table 1 for parameters not listed here.

$^c$ Values fit to experiment data
Table 4. Model parameters used to obtain Fig. 10*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>( D_s ) (cm(^2)/s)</td>
<td>( 1.2 \times 10^{-10} ) c</td>
<td>( x_{0,p} )</td>
<td>0.393 ( e )</td>
</tr>
<tr>
<td>( k_{p,d} ) (A/cm(^2))/(mol/cm(^3))(^{1.5} )</td>
<td>0.25 ( e )</td>
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</tr>
</tbody>
</table>

Parameters used in Eq. 15

<table>
<thead>
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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>C/66(^\dagger)</td>
<td>35</td>
<td>0.95</td>
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</tr>
<tr>
<td>C/13</td>
<td>28</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>C/7</td>
<td>21</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>C/2.7</td>
<td>13</td>
<td>0.88</td>
<td></td>
</tr>
<tr>
<td>C/1.3</td>
<td>8</td>
<td>0.8</td>
<td></td>
</tr>
</tbody>
</table>

* Refer to Table 1 for parameters not listed here.

Table 5. Correlations of parameters \( a \) and \( b \) for symmetry factor with current rate \( (x_c) \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values with 95% confidence intervals</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a = a_1 / (x_c + a_2) )</td>
<td></td>
</tr>
<tr>
<td>( a_1 )</td>
<td>7.453 ± 0.841</td>
</tr>
<tr>
<td>( a_2 )</td>
<td>0.1969 ± 0.0285</td>
</tr>
<tr>
<td>( b = b_1 x_c + b_2 )</td>
<td></td>
</tr>
<tr>
<td>( b_1 )</td>
<td>-0.2035 ± 0.014</td>
</tr>
<tr>
<td>( b_2 )</td>
<td>0.9536 ± 0.0054</td>
</tr>
</tbody>
</table>
Fig. 1 Q. Zhang, et al.

Cone shaped Li foil is stuck to current collector and placed just above the LiCoO$_2$ electrode as a reference electrode.

Fig. 1 Schematic of LiCoO$_2$ half cell setup used in the experiment. The reference electrode is used to measure the potential of the LiCoO$_2$ electrode.

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Fig. 2 Q. Zhang, et al.

Fig. 2 Experimental galvanostatic charge profiles of the LiCoO₂ half-cell (C=4mA).
Fig. 3 Q. Zhang, et al.

![Graph of potential vs. x in Li$_2$CoO$_2$ showing discharge profiles.]

Fig. 3 Experimental galvanostatic discharge profiles of the LiCoO$_2$ half-cell (C=4mA).
Fig. 4 Q. Zhang, et al.

Fig. 4 The electrochemical behavior of the LiCoO$_2$ electrode is different during charge from that during discharge. The overpotential increases with $x$ during galvanostatic discharge, but does not vary much during galvanostatic charge (C=4mA).
Fig. 5 Schematic of a LiCoO₂ half cell sandwich, consisting of LiCoO₂ cathode, separator and lithium foil anode.
Fig. 6 Q. Zhang, et al.

Fig. 6 The pseudo-2D model predicts an incorrect potential profile at the beginning of charge, especially from $x = 1.0$ to $x = 0.8$. Symbols represent experimental results. Line represents the simulation result.
Fig. 7 Q. Zhang, et al.

![Graph showing exchange current density vs. cell SOC]

Charge process ends at x=0.4

**Fig. 7** Change of exchange current density $i_{0,p}$ with cell SOC when simulating Fig. 6.
Fig. 8 Q. Zhang, et al.

Fig. 8 Comparison of experiment (symbols) and simulated (lines) charge profiles at different current rates. The rates from bottom to top: C/66, C/13, C/7, C/2.7, and C/1.3 (C=4mA).
Fig. 9 The best simulation results (lines) obtained when model parameters are constants.

The rates from top to bottom are C/66, C/13, C/7, C/2.7 and C/1.3 (C=4 mA).
Fig. 10 Comparison of experiment (symbols) and simulation (lines) discharge profiles at different current rates. Symmetry factor $\beta$ is assumed to decrease with cell SOD in galvanostatic discharge.
Fig. 11 The correlation of parameter $a$ with current rate.
Fig. 12 Q. Zhang, et al.

Values shown in Table 4
Correlation with current rate

Fig. 12 The correlation of parameter $b$ with current rate.

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Fig. 13 Q. Zhang, et al.

Fig. 13 Model predicted charge and discharge profiles for the LiCoO$_2$ electrode (C=4mA).

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Fig. 14 Q. Zhang, et al.

![Graph showing capacity vs. C rate]

Fig. 14 Comparison of experiment and model predicted charge/discharge capacity.

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Fig. 15 Ragone plot of the LiCoO$_2$ electrode.
Fig. 16 Calculated solution phase concentration distribution at 2C discharge rate.
Fig. 17 Q. Zhang, et al.

Fig. 17 Calculated solution phase potential distribution at 2C discharge rate.
Fig. 18 Q. Zhang, et al.

![Graph showing calculated solid phase concentration profiles in the half cell at 2C discharge rate.](image)

Fig. 18 Calculated solid phase concentration profiles in the half cell at 2C discharge rate.